Mittuniversitetet
MID SWEDEN UNIVERSITY

Försättsblad Prov Original

Kurskod	Provkod	Tentamensdatum
MA095G	T100	2018-06-11

Kursnamn	Matematik GR (A), Diskret matematik A
Provnamn	Tentamen
Ort	Sundsvall
Termin	V18
Ämne	Matematik

MID SWEDEN UNIVERSITY

MOD

Examination 2018

MA095G & MA098G Discrete Mathematics (English)

Time: 5 hours

Date: 11 June 2018

Pia Heidtmann

The compulsory part of this examination consists of 7 questions. The maximum number of points available is 24. The points for each part of a question are indicated at the end of the part in [ ]-brackets.

The final grade on the course is determined by how well the candidates demonstrate that they have met the learning outcomes on the course. Provided all learning outcomes have been met, the following guide values will be used to set the course grade:

E: 9p D: 10p C: 14p B: 18p A: 22p

The final question on the paper is the Aspect Question, it is optional and carries no value in terms of marks, but a good solution of this Aspect Question may raise a candidates grade by one grade.

The candidates are advised that they must always show their working, otherwise they will not be awarded full marks for their answers. The candidates are further advised to start each of the nine questions on a new page and to clearly label all their answers.

This is a closed book examination. No books, notes or mobile telephones are allowed in the examination room. Note that Mathematical Formula Collection Edition 4 is allowed on this tenta and will be available in the examination room.

Electronic calculators may be used provided they cannot handle formulas. The make and model used must be specified on the cover of your script.

GOOD LUCK!!

Question 1

(a) Consider the positive integer $s = \sum_{n=4}^{8} 2^{2n-3}$.

(i) Express s as a sum without using Σ-notation.

(ii) Give s in base 2 and base 8. [1.5p]

(b) Let $M = \{d, i, \{m\}, a, t, h, s, A\}$. Justifying your answers, say whether the following are true or false.

(i) $\{m\} \in M$;

(ii) $\{m\} \subseteq M$;

(iii) $\{m\} \subseteq \mathcal{P}(M)$;

(iv) $\{m\} \cup M = M$. [1p]

(c) How many subsets of the set $M = \{d, i, \{m\}, a, t, h, s, A\}$ contain less than 7 elements? [0.5p]

Question 2

(a) Let a, b and n be integers, where $n > 100$. Suppose that $13a \equiv 13b \pmod{n}$. Justifying your answer, say whether $a \equiv b \pmod{n}$ also. [0.5p]

(b) Compute $[2] \odot [3]^{-1}$ in $\mathbb{Z}_{19}$. [0.5p]

(c) Find all solutions $[x] \in \mathbb{Z}_{967}$ to the equation $[41] \odot [x] = [1]$ in $\mathbb{Z}_{967}$. [2p]

Question 3 Let the functions $h : \mathbb{N} \to \mathbb{Z}$ and $g : \mathbb{Z} \to \mathbb{Z}$ be given by the rules

$$h(x) = x^2 - 4 \quad \text{and} \quad g(x) = (x+3)^7.$$

(a) Prove that h is one-to-one. [1p]

(b) Show that h is *not* onto. [0.5p]

(c) Showing all your working and reducing your expression as much as possible, give the function rule for $g \circ h$. [1.5p]

(d) Show that $g \circ h$ is *not* $O(x)$. [1p]

Question 4

(a) State and prove *the pigeonhole principle.* [1.5p]

(b) Suppose we randomly select a subset of integers from the set

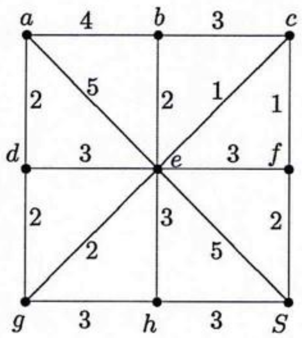
$$\{1, 2, 3, 4, \ldots, 200\}.$$

How large must the subset be, if we want to make sure that we have chosen at least one pair of integers a, b such that $31|(13a - 13b)$? Justify your answer. [1.5p]

Question 5

(a) Construct three non-isomorphic simple graphs with degree sequence $1, 2, 2, 2, 2, 3$ and explain why your three graphs are not isomorphic. [2p]

(b) Showing all your working, use *Dijkstra's algorithm* to find a shortest path from vertex a to vertex S in the weighted graph below. Take care to show how the vertices become labelled and how the labels change during the run of the algorithm. Finally, give the length of the shortest path. [2p]



Question 6

Let R_1 and R_2 be the two relations defined on the set $S = \{1, 2, 3, 4, 5, 6\}$ by
$(a, b) \in R_1$ if and only if $a|b \wedge b|a$, and
$(a, b) \in R_2$ if and only if $a|b \vee b|a$.

(a) (i) Draw the relation digraph for R_1.

(ii) Draw the relation digraph for R_2. [1p]

(b) (i) Is the relationen R_1 an equivalence relation on S? Either prove that it is, giving all equivalence classes, or give reasons why it is not an equivalence relation.

(ii) Is the relationen R_2 an equivalence relation on S? Either prove that it is, giving all equivalence classes, or give reasons why it is not an equivalence relation. [3p]

Question 7

A sequence is given by the recurrence relation

$$x_{n+1} = x_n + 3n \text{ for } n \geq 1,$$

and the initial value $x_1 = 1$.

(a) Showing your working, use the recurrence relation to compute x_n for $n = 1, 2, 3$ and 4. [1p]

(b) Prove for all $n \geq 1$ that $x_n = \dfrac{3n^2 - 3n + 2}{2}$. [2p]

Aspect Question

Let p be a positive prime.
Suppose that a and b are positive integers such that p divides the product ab.
Show that $p|a$ or $p|b$.

MITTUNIVERSITETET

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Tentamen 2018

MA095G & MA098G Diskret matematik (svenska)

Skrivtid: 5 timmar

Datum: 11 juni 2018

Pia Heidtmann

Den obligatoriska delen av denna tenta omfattar 7 frågor. Delfrågornas poäng står angivna i marginalen inom [ ]-parenteser. Maximalt poängantal är 24.

Betyg sätts efter hur väl lärandemålen är uppfyllda. Riktvärde för betygen är:

E: 9p D: 10p C: 14p B: 18p A: 22p

Därutöver innehållar skrivningen en frivillig aspektuppgift som kan höja betyget om den utförs väl med god motivering.

Behandla högst en uppgift på varje papper!

Till alla uppgifter skall fullständiga lösningar lämnas. Resonemang, ekvationslösningar och uträkningar får inte vara så knapphändiga, att de blir svåra att följa. Brister i framställningen kan ge poängavdrag även om slutresultatet är rätt!

Hjälpmedel: Matematisk Formelsamling Ed. 4 (delas ut), skriv- och ritmaterial samt miniräknare som ej är symbolhanterande. Ange märke och modell på din miniräknare på omslaget till tentamen.

LYCKA TILL!!

Uppgift 1

(a) Låt positiva heltalet $s = \sum_{n=4}^{8} 2^{2n-3}$.

(i) Ange s som en summa utan summatecken.

(ii) Uttryck talet s i basen 2 och basen 8. [1,5p]

(b) Låt $M = \{d, i, \{m\}, a, t, h, s, A\}$. Ange om följande påståenden är sanna eller falska. Motivera dina svar.

(i) $\{m\} \in M$;

(ii) $\{m\} \subseteq M$;

(iii) $\{m\} \subseteq \mathcal{P}(M)$;

(iv) $\{m\} \cup M = M$. [1p]

(c) Hur många delmängder till mängden $M = \{d, i, \{m\}, a, t, h, s, A\}$ innehåller mindre än 7 element? [0,5p]

Uppgift 2

(a) Låt a, b och n vara heltal där $n > 100$ och antag att $13a \equiv 13b \pmod{n}$. Är $a \equiv b \pmod{n}$? Motivera ditt svar! [0,5p]

(b) Beräkna $[2] \odot [3]^{-1}$ i $\mathbb{Z}_{17}$. [0,5p]

(c) Bestäm alla lösningar $[x] \in \mathbb{Z}_{967}$ till ekvationen $[41] \odot [x] = [1]$ i $\mathbb{Z}_{967}$. [2p]

Uppgift 3 Låt $h : \mathbb{N} \to \mathbb{Z}$ och $g : \mathbb{Z} \to \mathbb{Z}$ vara funktionerna definierade genom

$$h(x) = x^2 - 4 \quad \text{och} \quad g(x) = (x+3)^7.$$

(a) Visa att h är injektiv. [1p]

(b) Visa att h *inte* är surjektiv. [0,5p]

(c) Bestäm en funktionsregel för $g \circ h$ och förenkla uttrycket så mycket som möjligt. [1,5p]

(d) Bevisa att $g \circ h$ *inte* är $O(x)$. [1p]