Mittuniversitetet
MID SWEDEN UNIVERSITY

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Kurskod	Provkod	Tentamensdatum
DT049A	T101	2019-01-09

Kursnamn	Datateknik AV, Sannolikhetslära och stokastiska processer
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Termin	
Ämne	

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Exam of Probability and Random Processes
DT049A

8:00 - 13:00, January 9, 2019

Contact: Patrik Österberg (010-142 86 14)

INSTRUCTIONS
This is an open book and closed notes exam.
Aids allowed:

1. Robert G. Gallager *Stochastic Processes: Theory for Applications*

All answers must be thoroughly explained.
Be careful and mark all your solution papers with both your identification number and which question you are answering. Write clearly.
Maximum points: 36
Good Luck!

1. **Random Variables**

   Let X be a random variable having the following probability distribution:

$$p_X(-1) = p_X(0) = 1/8$$
$$p_X(1) = 1/4$$
$$p_X(2) = 1/2$$

   (a) (1 point) Calculate the expected value E[X].

   (b) (2 points) Calculate the variance σ_X^2.

   (c) (2 points) Two random variables are uncorrelated if their covariance $\sigma_{XY} = E[(X - E[X])(Y - E[Y])]$ is zero. Show that this is equivalent to $E[XY] = E[X]E[Y]$.

Total for Question 1: 5

2. **RV and Poisson Processes.** Let $Z : \Omega \to \mathbb{R}$ be a continuous random variable with PDF $f_Z(x)$. Now consider the transformation $Y = |Z|$, then:

   (a) (3 points) Prove that Y is still a random variable.

   (b) (2 points) Consider now a generic Poisson process with rate λ and let X be the random variable representing the interarrival time of the process. Write the PDF and the CDF of X and motivate your answer.

   (c) (2 points) Show that $|X|$ is also a random variable and that it is still exponentially distributed.

Total for Question 2: 7

3. **Finite-State Markov Chains.** Consider the following matrices:

$$P = \begin{bmatrix} 0 & 1/2 & 1/2 & 0 \\ 3/4 & 1/4 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad Q = \begin{bmatrix} 0 & 1/2 & 1/2 \\ 0 & 1/3 & 2/3 \\ 0 & 1/2 & 1/2 \end{bmatrix} \quad T = \begin{bmatrix} 1/10 & 1/2 & 2/5 \\ 1/2 & 1/4 & 1/4 \\ 3/10 & 2/5 & 1/2 \end{bmatrix} \quad R = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

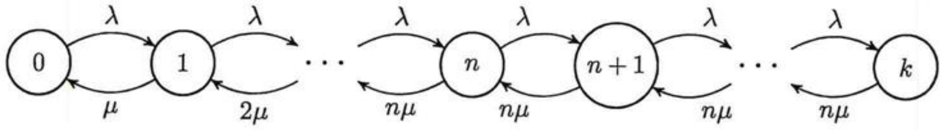
(a) (2 points) For each matrix, write if it is a transition probability matrix and if not explain why.

(b) (2 points) For each transition probability matrix, draw the associated Markov Chain.

(c) (3 points) Obtain the steady state probabilities of the Markov Chains.

Total for Question 3: 7

4. **Markov Processes.** Consider a M/M/n/k queue, with n servers and capacity k ($k \geq n$). Assume a generic arrival rate (λ) and a generic service rate per server (μ). Let $\rho = \lambda/\mu$. The Markov process for an M/M/n/k queue is presented in figure:



(a) (3 points) Let $n = 1$ and $k = 4$, find the expression for the time-average fraction of time for which the queue is full (k customers are in the system)

(b) (3 points) Now, let $n = k = 4$, find the new expression for the time-average fraction of time for which the queue is full (k customers are in the system)

(c) (2 points) Compare the fraction of time for which the queue is full obtained in point a) and b). Which one is larger? Explain why.

Total for Question 4: 8

5. **Decision Theory.** A binary sequence is used to generate a modulated signal X, which has amplitude A when a bit of the sequence is 1 and 0 when a bit of the sequence is 0. The signal is transmitted over an additive white Gaussian noise (AWGN) communication channel with noise distribution $N \sim \mathcal{N}(0, \sigma_n^2)$. Consider now that the behaviour of the source is known. In particular we know that i) each bit of the sequence is a i.i.d. random variable and, ii) the probability of generating a 0 is two times the probability of generating a 1. The transmitted message is then recovered (bit-by-bit decision) from the received noisy signal $Y = X + N$ using a MAP decision approach which takes into account both i) and ii).

(a) (2 points) Is the suggested hypothesis-testing approach Bayesian or non-Bayesian? Motivate your answer.

(b) (2 points) Write the *a priori* probability distribution of the hypothesis random variable X.

(c) (3 points) Calculate the optimal (MAP) decision split-point for signal detection at the receiver.

(d) (2 points) What would be the effect of ignoring the a priori distribution of the source bits? Discuss how the lack of this information would affect both the design and the performance of the receiver.

Total for Question 5: 9