<table>
<thead>
<tr>
<th>Kurskod</th>
<th>Provkod</th>
<th>Tentamensdatum</th>
</tr>
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<tbody>
<tr>
<td>ET027G</td>
<td>T101</td>
<td>2019-03-18</td>
</tr>
</tbody>
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<table>
<thead>
<tr>
<th>Kursnamn</th>
<th>Provnamn</th>
<th>Ort</th>
<th>Termin</th>
<th>Ämne</th>
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<tbody>
<tr>
<td>Elektroteknik GR (B), Kraftelektronik</td>
<td>Tentamen - Sundsvall</td>
<td>Sundsvall</td>
<td>VT2019</td>
<td>Elektroteknik</td>
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</tbody>
</table>
Power Electronics Exam (ET027G)

Date and Time: 18th March 2019 and 8:00 – 12:00
Responsible Teacher: Sobhi Barg, Tel: 010-142 79 91, Email: sobhi.barg@miun.se

Maximum points: 55
Preliminary grades: A≥ 53pt, B≥ 48pt, C≥ 43pt, D≥ 38pt, E≥ 33pt, F< 28pt

Introduction:

- The exam includes two answer sheets, please return them with your answer sheets.
- The exam includes 4 parts, which are completely separate.
- The first part contains Yes or No type Questions. Full mark will be given for correct answer and zero for incorrect answer.
- The second part is a problem about the full-bridge rectifier.
- The third part is a problem about the boost converter.
- The fourth part is a problem about the three phase inverter.
- Each numerical result has to be preceded by full and clear development of the analytical expression.
- The exam includes a formula sheet.
- The use of scientific calculator is allowed.
- English to Swedish dictionary is allowed.
I. Part One (15 pts)

Complete the following sentences by Yes or No

- In switching mode power supply, the transistor is operated in the active region: .......... 
- The power Bjt transistor has better switching capability than the power MOSFET: .......... 
- The conduction loss depends on the switching frequency: .......... 
- At high frequency, the switching loss are higher than the conduction loss: .......... 
- The power MOSFET is a current controlled device: .......... 
- The maximum output voltage of the full bridge diode rectifier is 207 V: .......... 
- The average output voltage of the three phase diode rectifier is .......... 
- The AC line input inductance increases the average output voltage of the rectifier: .......... 
- The harmonics currents are part of the active power: .......... 
- The output voltage of a buck converter is dependent on the load current: .......... 
- In buck converter, the inductance is proportional to the output current ripple: .......... 
- For DC-DC converters, the inductance for DCM is bigger than that for CCM: .......... 
- In flyback converter, the magnetic core operates in the first and third quadrant: .......... 
- The Mosfet voltage stress in boost converter is higher than that in flyback: .......... 
- The ZVS can be realized during the turn-off: .......... 

Note: Please include this page with your exam answer sheet.
II. Part Two (16pts)

Consider the single phase semi-controlled rectifier below. V_s is the AC line voltage V_s=230 V, f=50 Hz.

1- Determine the on-state and off-state conditions of each semi-conductor components in the circuit. (1pt)
2- Draw in the same figure, V_s, I_s, V_d for the firing angle α=45°, 90°. (1pt)
3- In the same figure but in different x-axis draw, the current and the voltage across each components (use different colors). (1pt)
4- Determine the expression of the average output voltage V_d and calculate its value for the different firing angles. (2pt)
5- Suppose that V_{dc} is the average output voltage when α=0°. Calculate the firing angle that gives V_d=0.5 * V_{dc}. (2pt)

Suppose that you have a dc voltage source (V_c= 60 V) in series with the high inductive load (id).

6- Draw the new circuit. (1pt)
7- What is the average output voltage when the firing angle is equal to 5° (suppose that the triggering pulse is very short (some nano-second). (2pt)
8- Determine the minimum firing angle to get an input voltage higher than V_c. (3pt)
9- Determine the firing angle to get an average output voltage V_d 30% higher than V_c. (3pt)
III. Part Three (16 pts)
Consider the circuit of a boost converter below. We would like to operate the converter in CCM. \( V_{\text{in}} = 2.3-8 \text{ V}, \ V_0 = 12 \text{ V}, \ f = 200 \text{ kHz}, \ P_o = 3.6 \text{ W}, \ \eta = 90 \%

![Boost Converter Diagram]

1- Explain the operation of this converter and draw the equivalent circuit for the two cases. (2pt)
   - The switch S is on.
   - The switch S is off.
2- Determine the minimum and maximum duty cycle D to get the required output voltage. (2pt)
3- Demonstrate that the expression of the output current ripple as function of \( V_0, D, L \) and \( f \) is equal to: (1pt)
   \[
   \Delta I_o = \frac{V_0 D (1 - D)}{L f}
   \]
4- Calculate \( \frac{\Delta I_o}{dD} = 0 \) and determine the critical duty cycle for which the current ripple is maximum. (3pt)
5- Calculate the critical input voltage that leads to the maximum output current ripple. (2pt)
6- Determine the required inductance to get an output current ripple equal to 20% the output DC current at the critical duty cycle. (2pt)
7- For the calculated inductance determine the minimum output current ripple that can be achieved. (2pt)
8- For \( D = 0.5 \), the calculated inductance and the given frequency, sketch in the same figure: the input voltage, the switch voltage \( V_{ds} \) in the first x-axis. In the second x-axis, draw the output current and the switch current. (2pt)
IV. Part four (8 pts)

Consider the three-phase inverter given in Fig.1. The 120° modulation scheme is used to drive the inverter as given in Fig.2. The 120 modulation scheme means that the gate signals are shifted by 120 and each switch conducts for 180°.

![Diagram of three-phase inverter](image)

Fig.1: three phase Voltage source inverter
1) Fill in the table below and complete the waveforms of $V_{AB}$, $V_{ON}$, and $V_{AN}$.

<table>
<thead>
<tr>
<th>C1</th>
<th>C2</th>
<th>C3</th>
<th>$V_{ON}$</th>
<th>$V_{AN}$</th>
<th>$V_{AB}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
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<tr>
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<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

Fig.2: gate signals of the upper switches using 120

Note: Please include this page with your exam answer sheet.
Formula sheet for Power Electronics, ET027G, Mid Sweden University

1) Constants

<table>
<thead>
<tr>
<th>Name</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vacuum Permeability</td>
<td>$\mu_0 = 4\pi \times 10^7$</td>
<td>$\frac{Vs}{A\pi m}$</td>
</tr>
<tr>
<td>Vacuum Permittivity</td>
<td>$\varepsilon_0 = 8.85 \times 10^{-12}$</td>
<td>$\frac{F}{\varepsilon}$</td>
</tr>
<tr>
<td>Speed of light in vacuum</td>
<td>$c = 2.9979 \times 10^8$</td>
<td>$\frac{m}{s}$</td>
</tr>
</tbody>
</table>

2) Passive components

<table>
<thead>
<tr>
<th>Resistor</th>
<th>Inductor</th>
<th>Capacitor</th>
</tr>
</thead>
<tbody>
<tr>
<td>$v = Ri$</td>
<td>$i = C\frac{dv}{dt}$</td>
<td>$Z = j\omega L$</td>
</tr>
<tr>
<td>$N\frac{\phi}{L}$</td>
<td>$\frac{\phi}{q}$</td>
<td>$v = \frac{q}{C}$</td>
</tr>
<tr>
<td>$Z = R$</td>
<td>$Z = j\omega L$</td>
<td>$Z = \frac{1}{j\omega C}$</td>
</tr>
<tr>
<td>$W = \frac{1}{2}Li^2$</td>
<td>$W = \frac{1}{2}Cv^2$</td>
<td></td>
</tr>
</tbody>
</table>

3) Instantaneous and Mean power

$P(t) = v(t)i(t)$

$P_{Avg} = \frac{1}{T} \int_0^T p(t)dt$

4) Efficiency

$\eta = \frac{P_{out}}{P_{in}} = 1 - \frac{P_{loss}}{P_{in}}$

5) Tank circuit resonance frequency

$f = \frac{1}{2\pi \sqrt{L/C}}$

6) Average value of the function

$F_{avg} = \frac{1}{T} \int_0^T f(t)dt$

7) RMS value of the function, $f(t)$, with harmonics $F_{1,2,\ldots}$:

$F_{rms} = \sqrt{\frac{1}{T} \int_0^T f^2(t)dt} = \sqrt{\sum_{n=1,2,\ldots} F_n^2} = \sqrt{F_1^2 + F_2^2 + F_3^2 + \ldots}$

8) Crest Factor

$CF = \frac{V_{peak}}{V_{RMS}}$

9) Some common waveforms (Observe peak value = 1 not RMS values as commonly used).

<table>
<thead>
<tr>
<th>Wave type</th>
<th>Waveform</th>
<th>Mean magnitude (rectified)</th>
<th>Waveforms factor</th>
<th>RMS values</th>
<th>Crest factor</th>
<th>Crest factor (dB)</th>
</tr>
</thead>
<tbody>
<tr>
<td>DC</td>
<td></td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0 dB</td>
</tr>
<tr>
<td>Sine wave</td>
<td>$\frac{2\pi}{2\sqrt{2}} \approx 1.11$</td>
<td>$\frac{1}{\sqrt{2}} \approx 0.707 \sqrt{\frac{1}{2}} \approx 1.414$</td>
<td>3.01 dB</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Full-wave rectified sine</td>
<td>$\frac{2\pi}{2\sqrt{2}} \approx 1.11$</td>
<td>$\frac{1}{\sqrt{2}} \approx 0.707 \sqrt{\frac{1}{2}} \approx 1.414$</td>
<td>3.01 dB</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Half-wave rectified sine</td>
<td>$\frac{1}{\pi} \approx 0.318$</td>
<td>$\frac{1}{2} \approx 0.5$</td>
<td>2</td>
<td>6.02 dB</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Triangle wave</td>
<td>$\frac{2}{\sqrt{3}} \approx 1.155$</td>
<td>$\frac{2}{\sqrt{3}} \approx 0.577 \sqrt{\frac{2}{3}} \approx 1.732$</td>
<td>4.77 dB</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Square wave</td>
<td>$\frac{1}{2} = 0.5$</td>
<td>$\frac{2}{\sqrt{3}} \approx 1.155$</td>
<td>$\frac{1}{\sqrt{3}} \approx 0.577 \sqrt{\frac{2}{3}} \approx 1.732$</td>
<td>0 dB</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

10) Power factor

$PF_{True} = \frac{P_{avg}}{S} = \frac{I_1}{I_s} PF_{Dipl} = \frac{1}{\sqrt{1 + THD^2}} PF_{Dipl} = PF_{Dis} PF_{Dipl}$
11) Analogy between Electric, Magnetic and thermal systems

<table>
<thead>
<tr>
<th>ELECTRICAL</th>
<th>THERMAL</th>
<th>MAGNETIC</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ohms law</td>
<td>Fourier's law</td>
<td>Hopkins</td>
</tr>
<tr>
<td>Current q</td>
<td>Power P</td>
<td>Flux Φ</td>
</tr>
<tr>
<td>Voltage V</td>
<td>Temperature T</td>
<td>mmf Ni</td>
</tr>
<tr>
<td>Electrical conductivity ρ</td>
<td>Thermal conductivity k λ</td>
<td></td>
</tr>
<tr>
<td>Electrical resistivity σ = 1/ρ</td>
<td>permeability μ</td>
<td></td>
</tr>
<tr>
<td>Capacitance C</td>
<td>Heat capacity C₀</td>
<td></td>
</tr>
<tr>
<td>Resistance R</td>
<td>Thermal resistance R₀</td>
<td>Reluctance W</td>
</tr>
</tbody>
</table>

12) RC series circuit

- τ = RC (time constant)
- \( f_c = \frac{1}{2πRC} \) (Cut off frequency)
- \( H_c(f) = \frac{1}{1+j2πRC} \) (Low pass)
- \( H_p(f) = \frac{1}{1+j2πRC} \) (High Pass)
- \( V_C(t) = V_0 + V(1 - e^{-t/τ}) \)
- \( V_R(t) = V_0 + V e^{-t/τ} \)

13) Total harmonic distortion, THD

\[ THD = \sqrt{\sum_{h=1}^{\infty} \left( \frac{I_{sh}}{I_{s1}} \right)^2} \]

14) Real Power in a three phase system:

\[ P = 3V_{ph}I_{ph} \cos φ = \sqrt{3}V_LI_L \cos φ \]

15) Magnetic reluctance

\[ R = \frac{l}{μA} \]

\[ R = \frac{l}{μ_0A} + \frac{l_x}{μ_0A_x} \] (with airgap)

16) Magnetic flux in one and two coupled coils

\[ \phi = \frac{mmf}{R} = \frac{NI}{R} \]

\[ \phi = \frac{NI_1 - NI_2}{R_1} = \frac{NI_1}{R_1} \]

17) Self inductance

\[ L = \frac{Nφ}{i} = \frac{N^2}{R} \]

18) RMS value of fundamental \( I_{s1} \) of a square wave with RMS value of \( I_s \). Note that the \( I_{rms} = \frac{I_{peak}}{\sqrt{2}} \) for square wave

\[ I_{s1} = \frac{2}{π} \sqrt{2}I_s = 0.9I_s \]

19) RMS value of Harmonic for a square wave signal with fundamental

\[ I_{sh} = \begin{cases} \frac{I_{s1}}{l} & \text{even } h \\ \frac{I_{s1}}{l} & \text{odd } h \end{cases} \]
Formula sheet for Power Electronics, ET027G, Mid Sweden University

20) Rectifier with constant current (large inductor) load and $I_d = 0$

Average diode voltage $V_{do} = \frac{2\sqrt{2}}{\pi} V_s = 0.9V_s$

Load current (constant) $I_d = \frac{2\sqrt{2}}{\pi} I_d = 0.9I_d$

21) Average load voltage of the single phase half wave rectifier for a sinusoidal input voltage with source inductance 'L_s'

$V_a = 0.45V_s - \frac{\omega L_s I_d}{2\pi}$

22) Average load voltage of the single phase full bridge rectifier for a sinusoidal input voltage with source inductance 'L_s'

$V_a = 0.9V_s - \frac{2\omega L_s I_d}{\pi}$

23) Commutation interval 'u' in a single phase half wave rectifier circuit with freewheeling diode

$\cos u = 1 - \frac{\omega L_s I_d}{\sqrt{2}V_s}$

24) Average load voltage of the three phase full bridge rectifier for a sinusoidal input voltage with source inductance 'L_s'

$V_a = 1.35V_{LL/(60\Omega)} - \frac{3}{\pi} \omega L_s I_d$

Commutation interval 'u' in a three phase full bridge converter circuit

$\cos u = 1 - \frac{2\omega L_s I_d}{\sqrt{2}V_{LL}}$

25) Commutation interval 'u' in a single phase thyristor based full bridge rectifier circuit with constant dc load current

$\cos(\alpha + u) = \cos(\alpha - \frac{2\omega L_s I_d}{\sqrt{2}V_s})$

26) Displacement power factor in a single phase thyristor based full bridge rectifier circuit with constant dc load current.

$DPF = \cos\left(\alpha + \frac{1}{2} u\right)$  \hspace{1cm} \alpha \text{ – Firing angle} \hspace{1cm} u \text{ – Time for current ramp due to } L_s \text{ in degrees}$

27) Average load voltage in a single phase thyristor based full bridge rectifier circuit with constant dc load current and source inductance $L_s$

$V_a = 0.9V_s \cos(\alpha - \frac{2\omega L_s I_d}{\pi})$
28) Displacement power factor in a three phase full bridge thyristor based rectifier for a sinusoidal input voltage

\[ \text{DPF} = \frac{1}{2} (\cos \alpha + \cos(\alpha + \pi)) \]

34) Switching losses

\[ P_{SW} \approx \frac{1}{2} I_{DS} V_{DS} (t_{on} + t_{off}) f_{SW} \]

\[ P_{Coss} = \frac{1}{2} C_{DS} V_{DS}^2 f_{SW} \]

35) Flyback converter

\[ \phi(t) = \phi(0) + \frac{V_d}{N_1} t \]

Core flux

\[ V_o = V_{in} \frac{N_2}{N_1} \frac{D}{1 - D} \]

Output voltage

\[ V_{sw} = V_{in} + \frac{N_1}{N_2} V_o = \frac{V_d}{1 - D} \]

Ideal Switch Stress

\[ V_{rect} = V_o + \frac{N_2}{N_1} V_{in} \]

Ideal Rectifier stress

36) Forward converter

\[ V_0 = V_{in} \frac{N_2}{N_1} D \]

37) Half Bridge Converter

\[ V_0 = V_{in} \frac{N_2}{N_1} D \]

38) Full Bridge Converter

\[ V_0 = 2V_{in} \frac{N_2}{N_1} D \]

29) Buck Converter in CCM

\[ V_o = V_{in} D \]

\[ \Delta V_o = \frac{\pi^2}{2} \left( 1 - D \right) \left( \frac{f_s}{f_s} \right)^2 = \frac{T_s}{8C} \frac{V_o}{L} (1 - D) T_s \]

30) Boost Converter in CCM

\[ V_o = \frac{V_{in}}{1 - D} \]

\[ \Delta V_o = \frac{V_o D T_s}{RC} \]

31) Buck-Boost Converter in CCM

\[ V_o = \frac{V_{in} D}{1 - D} \]

\[ \Delta V_o = \frac{V_o D T_s}{RC} \]

32) Gate drive power

\[ P_{Gate} = V_{gs} Q_b f \]

33) Conduction losses

\[ P_c = R_{DS(on)} I_{BRMS}^2 \]