Mittuniversitetet
MID SWEDEN UNIVERSITY

Försättsblad Prov Original

Kurskod	Provkod	Tentamensdatum
MA095G	T100	2019-03-20

Kursnamn	Matematik GR (A), Diskret matematik A
Provnamn	Tentamen - Sundsvall
Ort	Sundsvall
Termin	VT2019
Ämne	Matematik

8982538474

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MID SWEDEN UNIVERSITY

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Examination 2019

MA095G & MA098G Discrete Mathematics (English)

Time: 5 hours

Date: 20 March 2019

Pia Heidtmann

The compulsory part of this examination consists of 8 questions. The maximum number of points available is 24. The points for each part of a question are indicated at the end of the part in [ ]-brackets.

The final grade on the course is determined by how well the candidates demonstrate that they have met the learning outcomes on the course. Provided all learning outcomes have been met, the following guide values will be used to set the course grade:

E: 9p D: 10p C: 14p B: 18p A: 22p

The final question on the paper is the Aspect Question, it is optional and carries no value in terms of marks, but a good solution of this Aspect Question may raise a candidate's grade by one grade.

The candidates are advised that they must always show their working, otherwise they will not be awarded full marks for their answers. The candidates are further advised to start each of the nine questions on a new page and to clearly label all their answers.

This is a closed book examination. No books, notes or mobile telephones are allowed in the examination room. Note that Mathematical Formula Collection Edition 5 is allowed on this tenta and will be available in the examination room.

Electronic calculators may be used provided they cannot handle formulas. The make and model used must be specified on the cover of your script.

GOOD LUCK!!

Question 1

(a) Consider the positive integer $s = \sum_{n=0}^{9} 8^n$.

(i) Give the sum s without using Σ-notation.

(ii) Express s in base 8 and base 16. [1.5p]

(b) Let $D = \{d, m, a, 2, 0, 1, 9\}$.
How many subsets of cardinality 3 are there in the set $\mathcal{P}(D)$?
Justify your answer! [1p]

Question 2

(a) Let the function $f : \mathbb{N} \to \mathbb{Z}$ be given by the rule

$$f(x) = x^2 - 4.$$

Justifying your answer, say whether f is one-to-one. [1p]

(b) Let the function $g : \mathbb{R} \to \mathbb{R}$ be defined by

$$g(x) = -2x^2 - 1.$$

(i) Compute $g \circ g(x)$.

(ii) Prove that the function $g \circ g$ is $O(x^4)$. [1,5p]

Question 3

(a) Let p, q, r_1 and r_2 be logical propositions. Give a truth table for each of the following two compound propositions and decide whether they are equivalent or not.

$p_1 : (p \wedge q) \Rightarrow (r_1 \vee r_2)$;
$p_2 : (\neg p \wedge \neg q) \Rightarrow (\neg r_1 \vee \neg r_2)$. [1.5p]

(b) Consider the following statement concerning an integer n.

If n is a prime and $4|2n$ then $n < 0$ or $n = 2$.

Write down the contrapositive of this statement. [1p]

Question 4
Showing your working, find all solutions $[x] \in \mathbb{Z}_{2019}$ of the equation
$[203] \odot [x] = [1]$ in $\mathbb{Z}_{2019}$. [2p]

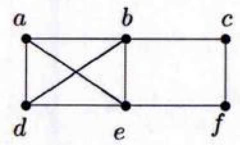
Question 5

(a) Is it possible to construct a simple graph with degree sequence

$$1, 1, 1, 1, 2, 2, 3, 3, 3, 4, 4?$$

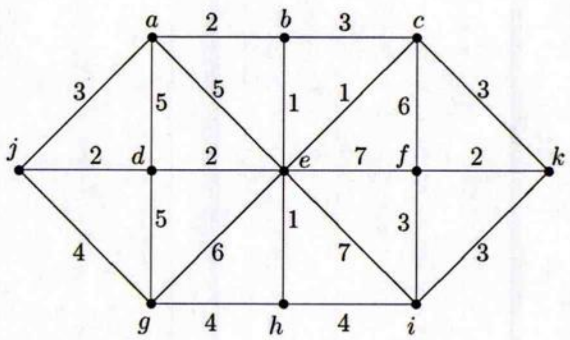
Either construct an example of such a graph or say why it is not possible to do so. [0.5p]

(b) Justifying your answer, find three non-isomorphic spanning trees in the following graph. [2p]



(c) Show that the graph from part (b) is not bipartite. [1p]

(d) Showing all your working, use *Kruskal's* or *Prim's* algorithm to find a minimum spanning tree in the following weighted graph, and give the total weight of the spanning tree found. [1.5p]



Question 6

(a) Let a and b be positive integers.
Give a proof by contradiction to show that if $5a = 2b$ then $2|a$. [1p]

(b) State and prove *the pigeonhole principle.* [1.5p]

(c) Suppose we randomly select a subset of integers from the set

$$\{10, 11, 12, 13, \dots, 50\}.$$

How large must the subset be, if we want to make sure that we have chosen at least one pair of integers a, b such that $2|(a-b)$ and $5|(b-a)$? Justify your answer. [1p]

Question 7
Let $S = \{0, \pm1, \pm2, \pm3, \dots, \pm8\}$ and consider the relation R on S defined by $(a, b) \in R$ if and only if $[a]_4 = [b]_4$ i $\mathbb{Z}_4$.

(a) Prove that R is an equivalence relation on S.

(b) Give the equivalence classes for the equivalence relation R on S. [3p]

Question 8

Consider the sum

$$s_n = \sum_{i=2}^{n} \binom{i}{2}$$

for any integer $n \geq 2$.

(a) Compute s_2, s_3, s_4 and s_5. [1p]

(b) Use induction to prove that $s_n = \binom{n+1}{3}$ for all $n \geq 2$. [2p]

Aspect Question
Prove that there exist irrational numbers $x, y \in \mathbb{R}$ such that the number y^x is rational.
(Hint: Consider the number $\left(\sqrt{2}\right)^{\sqrt{2}}$)

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Tentamen 2019

MA095G & MA098G Diskret matematik (svenska)

Skrivtid: 5 timmar

Datum: 20 mars 2019

Pia Heidtmann

Den obligatoriska delen av denna tenta omfattar 8 frågor. Delfrågornas poäng står angivna i marginalen inom [ ]-parenteser. Maximalt poängantal är 24.

Betyg sätts efter hur väl lärandemålen är uppfyllda. Riktvärde för betygen är:

E: 9p D: 10p C: 14p B: 18p A: 22p

Därutöver innehållar skrivningen en frivillig aspektuppgift som kan höja betyget om den utförs väl med god motivering.

Behandla högst en uppgift på varje papper!

Till alla uppgifter skall fullständiga lösningar lämnas. Resonemang, ekvationslösningar och uträkningar får inte vara så knapphändiga, att de blir svåra att följa. Brister i framställningen kan ge poängavdrag även om slutresultatet är rätt!

Hjälpmedel: Matematisk Formelsamling Ed. 5 (delas ut), skriv- och ritmaterial samt miniräknare som ej är symbolhanterande. Ange märke och modell på din miniräknare på omslaget till tentamen.

LYCKA TILL!!
