Goldilocks’ approach to mathematics teacher education:

WHAT DOES IT TAKE TO LEARN TO TEACH TO REASON?

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Forzani (2014) – a core practices approach (emphases added)

ambitious learning goals [...] grounded in the expectation that all students will develop high-level thinking, reasoning, and problem-solving skills

a partially improvisational practice, contingent on the ideas and contributions that are offered

making [...] subject-matter [...] a critical component of the goals and activities that constitute the professional curriculum

Today’s programme

1: Setting the scene:
• Background: a reform and the role of the teacher in it;
• Two premises and a conclusion
2: Why problematic?
3: A new conclusion – balancing proving that and proving why.

Larry’s grade 5: perfect squares and cubes

3² = 9
5² = 25
6² = 36
8² = 64

3³ = 9
4³ = 64
6³ =
8³ =

Steven: “Between each number there is an increase of two”.

Larry’s grade 5: perfect squares and cubes

| n | 1 | 2 | 3 | 4 | 5 | ...
|---|---|---|---|---|---|---
| n² | 1 | 4 | 9 | 16 | 25 | ...

3
5
7
9
11

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1: Setting the scene:
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Background: the teacher in the reform

Curricular frameworks and guides, instructional materials, and lesson plans are only the first elements needed to help students learn important mathematics well. Teachers must balance purposeful, planned classroom teaching with the ongoing decision-making that can lead the teacher and the class into uncharted territory from an effective mathematical and pedagogical knowledge base. (NCTM, 1998, p. 33)

Premise 1: R&P in school: not easy; should focus on proving why

... the plight of the legions of students who never appropriate the procedures of formal mathematical proof [...] who commit the proofs of standard theorems to memory in order (hopefully) to reproduce them in exams but who are unable to see the architecture of the arguments [...] who despair of constructing their own proofs, who dread and avoid exam questions that include the injunction "Prove that...".

Rowland, 2002, p. 158

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RaFiTE – pilot (DK): Cognitive results: Why is the sum of two odd numbers even? (n=57; 54 answers)

Twelve (almost) acceptable svar:

"It is in the definition of an odd number that it has remainder of 1 when divided by 2. Two odd numbers with the remainder of 1 that are added have the remainder of 2 -> no remainder"

"Because one number does not have someone to hold hands with, it gets one from the other, who also does not have a partner"
Eleven empirical arguments:

“Adding two odd numbers we will always get an even number, e.g. 9+9=18, 11+11=22, 17+17=34.”

I would try with some examples. 1+1=2, 3+3=6, 7+7=14. […] Okay, I have no idea how to prove this. (I am one of those students who never asked why, but just accepted because it made sense to me)”

RaPiTE – pilot (DK): Cognitive results: Why is the sum of two odd numbers even (n=57; 54 answers)

Others:

“Because the result if divided by 2 is a whole number”

“Tell them about the hierarchy of –, + and division”

Reasoning and proof (NCTM, 2008)

Proving why, e.g. using generic arguments

1. Gauss: 1+2+3+ … +100 = …

2. Larry and the difference between two consecutive perfect squares.

Larry’s grade 5: perfect squares

<table>
<thead>
<tr>
<th>n</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>…</th>
</tr>
</thead>
<tbody>
<tr>
<td>n²</td>
<td>1</td>
<td>4</td>
<td>9</td>
<td>16</td>
<td>25</td>
<td>…</td>
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</tbody>
</table>

5² − 4² = 4 + 5

(n+1)² − n² = n + (n+1)
Proving why, e.g. using generic arguments

1. Gauss: $1+2+3+\ldots+100 = \ldots$

2. \ldots

3. Larry and the difference between two consecutive perfect squares.

The generic argument requires one to think of the general while operating on the specific.

May support student understanding of the contents of the proof and the idea of proving in mathematics.

Premise 1: School mathematics should focus on proving why (e.g. generic arguments)

Premise 2: TE should focus on school mathematics

Conclusion: TE should focus on proving why

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Reasoning and proof (NCTM, 2008)
Larry’s grade 5: perfect squares

9² - 7² = 32  8² + 6² = 100  12² - 6² = 108
5² + 8² = 89 ...

9² - 1 = 80  3² - 1 = 8  7² - 1 = 48
5² - 1 = 24 ...

'This is all in the 8-times table'

Ahmed’s Pythagorean triples

<table>
<thead>
<tr>
<th>n</th>
<th>2p+1</th>
<th>n²</th>
<th>n² - 1</th>
<th>4p² + 1</th>
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</thead>
<tbody>
<tr>
<td>1</td>
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<td>21</td>
<td>42</td>
<td>441</td>
<td>400</td>
<td>800</td>
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</tbody>
</table>

1: \((2p+1)^2 - 1 = 4p^2 + 4p = 4(p+1)^2\). If \(p\) is even, 8 divides \(4p\); if \(p\) is odd, \((p+1)\) is even ...

2: \((n^2-1) = (n+1)(n-1)\). This is the product of two consecutive even numbers ...

3: Induction: If \(n^2-1 = 8q\), then \((n+2)^2-1 = n^2+4n+4 - n^2+1 = 4n+3 = 8q + 4 \Rightarrow 2(n+1)^2 = 8q + 4\), \(n+1\) is even ...

4: The difference between two consecutive numbers of this kind: \((n+2)^2 - 1 = n^2+4n+4\). If \(n\) is odd, \(n \equiv 1, 3, 5, 7 \pmod{8}\). Then \(n^2 \equiv 1 \pmod{8}\), and ...

Take an odd number and square it.

Divide by 2 and round up/down. Then I had two whole numbers next to one another.

'They are all in the 8-times table'

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Reasoning and proving in MTE

Striking a balance between proving why and proving that;
The balance requires MTE to be “sufficiently close” to school and “sufficiently close” to the discipline of mathematics;
This means:
• Starting off with challenges of R&P in school;
• Taking proving why as far as possible;
• Establish proving that as a related cultural activity;

A different interpretation of tradition-based teacher education.

References


